Blacktown Boys' High School

2023 Year 12

HSC Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions in Section II, show all relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 8)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 35)
- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section

Assessor: X. Chirgwin

Student Name:_____

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2023 Higher School Certificate Examination.

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

1 Which of the following is a many-to-one relation?

A. $x^2 + y^2 = 4$

B. $x = y^2 + 4$

C. $y = |x - 4|$

D. $y = 4e^x$

2 An infinite geometric series has a first term of 15 and a limiting sum of 10.5.
What is the common ratio?

A. $\frac{7}{10}$

B. $-\frac{7}{10}$

C. $\frac{3}{7}$

D. $-\frac{3}{7}$

3 In a normally distributed set of scores, the mean is 61 and the standard deviation is 8.
Approximately what percentage of the scores will lie between 53 and 85?

A. 83.85%

B. 81.5%

C. 65.7%

D. 49.85%

4 $\int \frac{9^x}{2} dx$ is equivalent to which of the following, where C is a constant?

A. $\frac{2 \times 9^x}{\ln 9} + C$

B. $\frac{9^x}{2 \ln 3} + C$

C. $\frac{9^x}{\ln 9} + C$

D. $\frac{9^x}{4 \ln 3} + C$

5 Sayem has six different pairs of socks. If two socks are selected at random, what is the probability that they will be a matching pair?

A. $\frac{1}{6}$

B. $\frac{1}{11}$

C. $\frac{1}{36}$

D. $\frac{1}{132}$

6 The function $y = \log_e x$ is transformed to $y = \log_e(3x - 3)$.

Which of the following describes the transformation that took place?

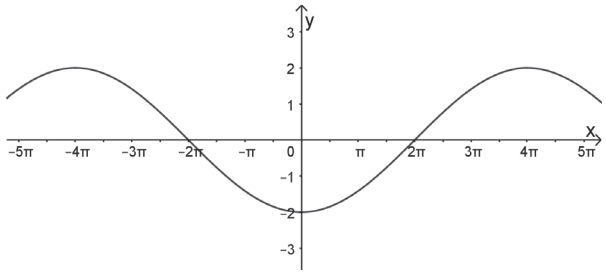
A. Horizontal dilation factor 3, followed by translation right 1 unit.

B. Horizontal dilation factor 3, followed by translation right 3 units.

C. Horizontal dilation factor $\frac{1}{3}$, followed by translation right 1 unit.

D. Horizontal dilation factor $\frac{1}{3}$, followed by translation right 3 units.

7 Which of the following is the correct equation for the graph shown?



A. $y = -2\sin\left(\frac{x}{4} + \frac{\pi}{2}\right)$

B. $y = 2\sin\left(\frac{x}{4} - \frac{\pi}{4}\right)$

C. $y = -2\sin\left(\frac{x}{2} + \frac{\pi}{2}\right)$

D. $y = 2\sin\left(\frac{x}{4} + \frac{\pi}{4}\right)$

8 Shiv invests money for three years at 2% per half year, compounded every half year.

Compound values of $1

Period	*Interest rate per period*			
	1%	2%	3%	4%
1	1.010	1.020	1.030	1.040
2	1.020	1.040	1.061	1.082
3	1.030	1.061	1.093	1.125
4	1.041	1.082	1.126	1.170
5	1.051	1.104	1.159	1.217
6	1.062	1.126	1.194	1.265

Using the table, which figure should Shiv use to calculate his investment?

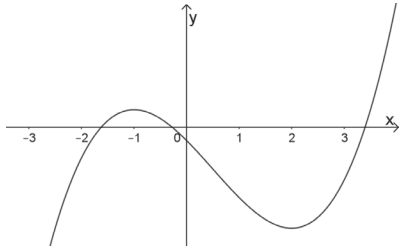
A. 1.020

B. 1.061

C. 1.126

D. 1.265

9 The diagram shows part of $y = f(x)$ which has a local maximum at $x = -1$ and a local minimum at $x = 2$.



Which of the following shows the correct relationship between $f'(-1)$, $f(0)$, and $f''(2)$?

A. $f(0) < f''(2) < f'(-1)$

B. $f(0) < f'(-1) < f''(2)$

C. $f''(2) < f'(-1) < f(0)$

D. $f''(2) < f(0) < f'(-1)$

10 It is known that $f(x)$ is an odd function and $g(x)$ is an even function. Given that $f(5) = -3$ and $g(3) = -5$, what is the value of $f\big(g(-3)\big) - g\big(f(-5)\big)$?

A. 15

B. 8

C. 2

D. -8

End of Section I

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Student Name: _____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

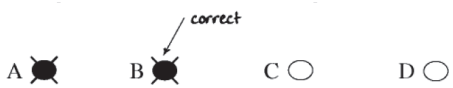
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.



Start Here ➔

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Mathematics Advanced

Section II Answer Booklet 1

90 marks
Attempt Questions 11 – 32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 22 (45 marks)
Booklet 2 – Attempt Questions 23 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Student Name: _____

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Question 11 (2 marks)

Differentiate $f(x) = \log_e(10x - 1) + \tan 5x$ with respect to x. **2**

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Question 12 (3 marks)

For the arithmetic sequences $6, 17, 28, 39, ...$

(a) What is the 13^{th} term? **2**

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(b) Find the sum of the first 21 terms. **1**

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Question 13 (2 marks)

The number of days, D, to complete a research project is inversely proportional to the number of researchers, R, who are working. The research project takes 123 days to complete when there are 7 people working on it. Find the equation relating D and R. **2**

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Question 14 (5 marks)

Given that $f(x) = \dfrac{1}{2x - 1}$ and $g(x) = 3x + 2$

(a) Find $f(g(x))$ in the simplest form. **2**

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(b) Find the domain of $f(g(x))$. **1**

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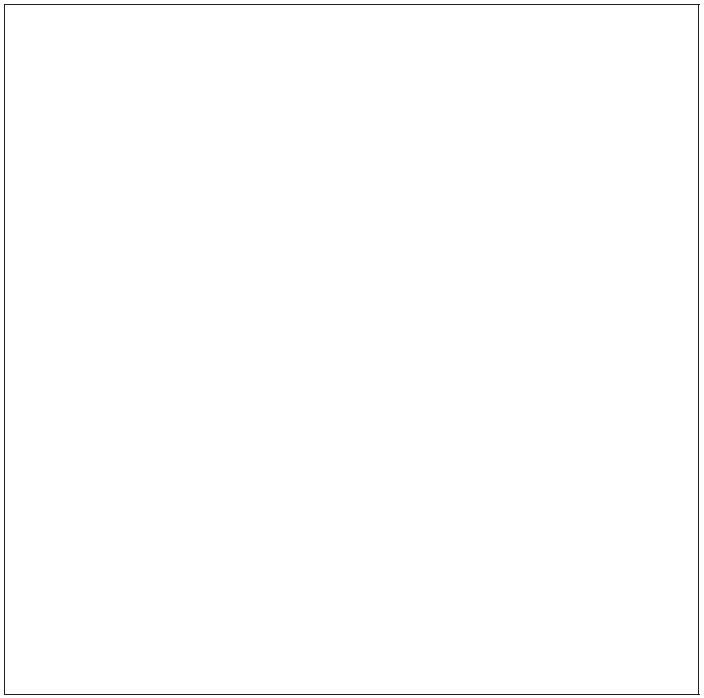
(c) Sketch $y = f(g(x)) + 1$, showing all intercepts and asymptotes. **3**



Question 15 (4 marks)

Saron and Rayaan take their driving tests on the same day. The probability of Rayaan passing his driving test is 0.65. The probability of both Saron and Rayaan passing is 0.533.

(a) What is the probability of Saron passing his driving test? **1**

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(b) What is the probability of both Saron and Rayaan failing their driving tests? **2**

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(c) What is the probability of at least one of them failing their driving tests? **1**

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Question 16 (3 marks)

Find the gradient of the normal to the curve $y = \frac{(4-x^2)}{e^{3x}}$ at the point $(0, 4)$. **3**

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Question 17 (4 marks)

Find

(a) $\int \left(6x^2 - \sin\frac{x}{3}\right) dx$ **2**

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(b) $\int_0^1 \frac{1}{(3-2x)^5} dx$ **2**

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Question 18 (3 marks)

The third and fifth terms of a geometric series are 21.6 and 31.104 respectively. Find the values of the first term and the common ratio. **3**

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Question 19 (5 marks)

The mean mass of a golden retriever dog is 34 kg and the standard deviation is 5 kg.

(a) What is the z-score of a 28 kg golden retriever dog? **1**

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(b) The recommended mass for golden retrievers is between 24 kg and 34 kg. **1**
If a golden retriever's mass has a z-score of 1.8, what is the minimum mass it needs to lose to lie within the healthy mass range?

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(c) It is known from statistical table that for this distribution approximately 79% of the golden retrievers have a mass less than 38 kg. **3**
What is the approximate percentage of golden retrievers with a mass between 30 kg and 44 kg?

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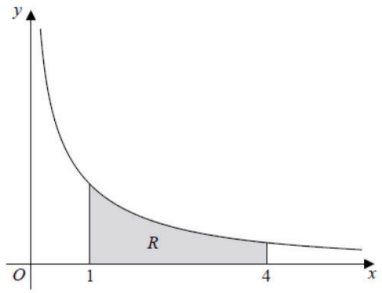
Question 20 (4 marks)

The following shows a sketch of part of the curve $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$.

The finite region R shown is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.



(a) The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$ **1**

Complete the table by giving the missing value of y to 4 decimal places.

x	1	2	3	4
y	1.4286	0.9033		0.5556

(b) Using the trapezoidal rule and the 4 function values in the table in part (a), find an estimate for the area of R, giving your answer to 4 decimal places. **2**

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(c) With reference to the curve, explain whether your estimate in part (b) is an overestimate or an underestimate for the area of R. **1**

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Question 21 (4 marks)

(a) Prove the identity $\frac{1}{\sin x + 1} - \frac{1}{\sin x - 1} = 2\sec^2 x$ **2**

(b) Hence find $\int_0^{\frac{\pi}{4}} \left(\frac{1}{3\sin x + 3} - \frac{1}{3\sin x - 3}\right) dx$ **2**

Question 22 (5 marks)

A bottle of vintage wine cost \$375 when first released. After t years its value, $\$V$, is given by $V = 375e^{0.05t}$.

(a) Find the value of the bottle of wine after 7 years, correct to the nearest dollar. **1**

(b) Find how many years it takes for the value of the wine to increase to \$1200 per bottle. Round your answer to 1 decimal place. **2**

(c) What is the rate of increase in the value of the wine 7 years after it was first released? Round your answer to 1 decimal place. **2**

End of Question 22
Proceed to Booklet 2

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Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 23 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

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Question 23 (5 marks)

One end of an elastic string was attached to a horizontal bar and a mass, m grams, was attached to the other end. The mass was suspended freely and allowed to settle.

The length of the string, l mm, was recorded, for various masses as follows.

m	100	200	300	400	500	600
l	228	236	256	278	285	301

(a) Calculate the Pearson's correlation coefficient, correct to two significant figures, and use it to describe the association between the mass and the length in terms of strength and direction. **2**

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(b) Determine the equation of the least-squares regression line of l in terms of m. Round your values to two significant figures. **2**

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(c) Calculate the mass attached to the string if the string length is 275 mm, correct to the nearest gram. **1**

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Question 24 (6 marks)

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

Present value of $1				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

Gurnoor borrows $22 000 for a car. He arranges to repay the loan with monthly repayments over 4 years. He is charged 7.8% per annum interest.

(a) Find Gurnoor's monthly repayment, correct to the nearest cent. **2**

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(b) Calculate the amount of interest Gurnoor will pay over the term of the loan, correct to the nearest cent. **2**

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Question 25 (4 marks)

A brand of rechargeable batteries provides power for 32 hours when first purchased fully charged. After its first recharge it only provides power for 30.4 hours. After its second recharge it only supplies power for 28.88 hours. Each subsequent recharging results in the battery having 95% of its previous power available.

(a) How many hours of power is available after the third recharge? Round to 2 decimal places. **1**

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(b) How many hours could you expect to get out of the battery? **1**

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(c) If the battery is thrown away when its charge level after recharging is less than two hours, how many times would it be recharged? **2**

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Question 26 (3 marks)

Care Now is a Nursing agency which specialises in sending nurses out to provide specialised home care. They have analysed the number of care request applications received over the last five years. They found that the number of applications (N), measured in hundreds, for specialised home care at time, t months, is given by the function

$$N = 3.6\sin\left(\frac{5\pi}{28}t\right) + 5.1$$

(a) Find the maximum and minimum number of applications. **2**

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(b) What is the time interval, in months, between two successive maximum number of applications? **1**

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(c) For how long between successive maximum number of applications will the number be at least 330? **3**

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Question 27 (2 marks)

The graph of $y = x^2$ is transformed into the graph of $y = 3x^2 + 24x + 33$ by the transformations: **2**

- A vertical stretch with scale factor k followed by
- A horizontal translation of p units followed by
- A vertical translation of q units

Write down the values of k, p and q.

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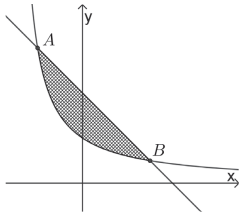
Question 28 (3 marks)

The diagram below shows the graphs of $y = \frac{3}{2x+3}$ and $x + y = 2$, which intersect at A and B. The x values of the coordinates at A and B are -1 and 1.5 respectively. **3**



Find the exact shaded area bounded by $y = \frac{3}{2x+3}$ and $x + y = 2$.

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Question 29 (5 marks)

The mass, X kg, of luggage taken on board an aircraft by a passenger can be modelled by the probability density function

$$f(x) = \begin{cases} kx^3(30 - x) & 0 \leq x \leq 30 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is $\frac{1}{1215000}$ **2**

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(b) Verify that the median mass of luggage is about 20.586 kg. **1**

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(c) Find the probability of a passenger taking on board luggage with a mass greater than 10 kg and less than 30 kg. Round the answer to 2 decimal places. **2**

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Question 30 (6 marks)

The function $g(x) = xe^{x+1}$ has first derivative $g'(x) = (1 + x)e^{x+1}$.

(a) Show that $g''(x) = (2 + x)e^{x+1}$ **1**

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(b) Find the coordinates of the stationary point and determine its nature. **2**

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(c) Find the coordinates of any point of inflection. **1**

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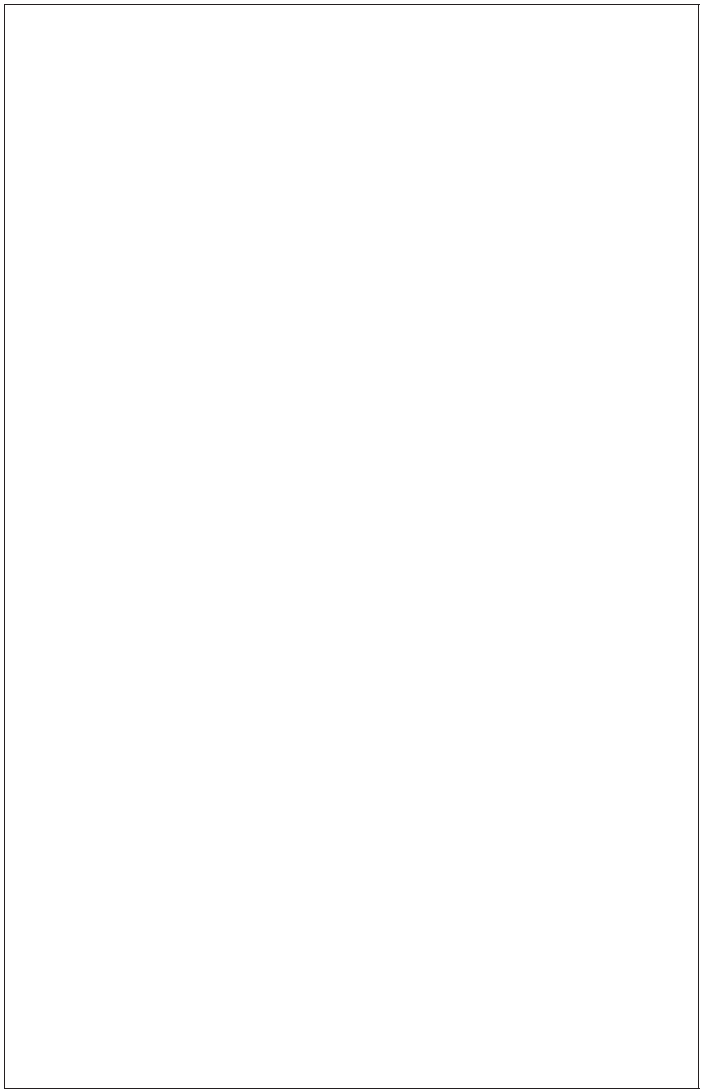
Question 30 continues on next page

Question 30 (continued)

(d) Sketch the curve $g(x) = xe^{x+1}$, showing the stationary point, the point of inflection and other key features. **2**



Question 31 (5 marks)

A particle moves on the x-axis with its velocity, v m/s, given at any time, t seconds, $t \geq 0$, by $v = \dfrac{6}{\sqrt{5t+4}}$

Initially the particle is at the origin.

(a) Find the initial velocity. **1**

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(b) Find the exact acceleration of the particle after 9 seconds. **2**

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(c) Find the displacement of the particle as a function of time. **2**

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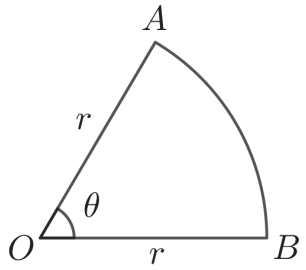
Question 32 (5 marks)

The diagram below shows a sector OAB of a circle with centre O and radius r cm. The arc AB subtends an angle θ radians at O.



(a) Show that the perimeter of the sector is $r(2+\theta)$. **1**

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(b) Given that the perimeter of the sector OAB is 22 cm, show that its area is **1**

$$A = \frac{242\theta}{(2+\theta)^2}$$

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Question 32 continues on next page

Question 32 (continued)

(c) Hence find the maximum area of the sector OAB. **3**

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Section II extra writing space

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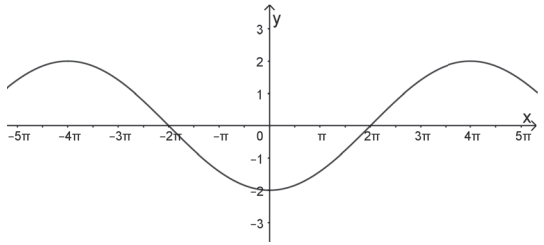
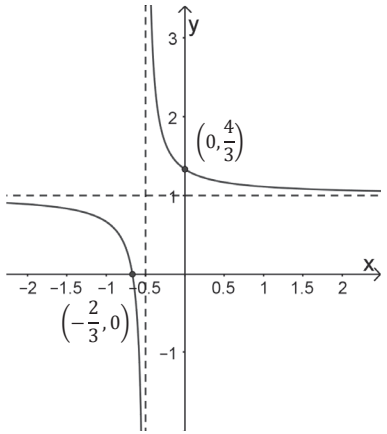
2023 Year 12 Mathematics Advanced AT4 Solutions

Section 1

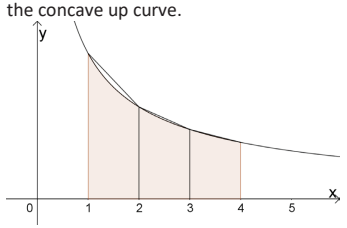
Q1	**C** Option A is a circle, this is a many-to-many relation. Option B is a side-ways parabola, this is an one-to-many relation. Option C is an absolute value function, this is a many-to-one relation. Option D is an exponential function, which is an one-to-one relation.	1 Mark
Q2	**D** $10.5 = \frac{15}{1-r}$ $1-r = \frac{15}{10.5}$ $r = 1 - \frac{10}{7}$ $r = -\frac{3}{7}$	1 Mark
Q3	**A** 53 is 1 sd below 61 and 85 is 3 sd above 61. $\frac{1}{2} \times 68\% + \frac{1}{2} \times 99.7\% = 83.85\%$	1 Mark
Q4	**D** $\int \frac{9^x}{2} dx$ $= \frac{1}{2} \times \frac{9^x}{\ln 9} + C$ $= \frac{9^x}{2 \ln 3^2} + C$ $= \frac{9^x}{4 \ln 3} + C$	1 Mark
Q5	**B** It doesn't matter which sock was chosen first, there is only one sock out of the left over 11 socks to make a matching pair.	1 Mark
Q6	**C** $y = \log_e x \rightarrow y = \log_e(3x)$ Horizontal dilation factor is $\frac{1}{3}$ $y = \log_e(3x) \rightarrow y = \log_e(3x-3) = \log_e\big(3(x-1)\big)$ Translation right 1 unit	1 Mark
Q7	**A**  $y = -2\sin\left(\frac{x}{4} + \frac{\pi}{2}\right)$	1 Mark

Q8	**C** 3 years compounded every half year → period is 6 2% with period 6 is 1.126	1 Mark
Q9	**B** $f'(-1) = 0, f(0) < 0, f''(2) > 0$ $f(0) < f'(-1) < f''(2)$	1 Mark
Q10	**B** $g(x)$ is even, $g(-3) = g(3) = -5$ $f(x)$ is odd, $f(-5) = -f(5) = 3$ $f\big(g(-3)\big) - g\big(f(-5)\big)$ $= f(-5) - g(3)$ $= 3 - (-5)$ $= 8$	1 Mark

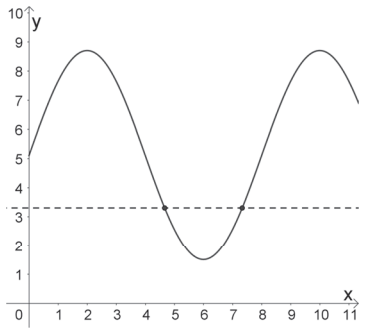
Section 2		
Q11	$f(x) = \log_e(10x - 1) + \tan 5x$ $f'(x) = \frac{10}{10x - 1} + 5\sec^2 5x$	2 Marks Correct solution 1 Mark Correct differentiation of $\log_e(10x - 1)$ or $\tan 5x$
Q12a	$a = 6, d = 11$ $T_{13} = 6 + (13 - 1) \times 11$ $T_{13} = 138$	2 Marks Correct solution 1 Mark Identifies d and correct substitution into the term formula
Q12b	$S_{21} = \frac{21}{2}(2 \times 6 + (21 - 1) \times 11)$ $S_{21} = 2436$	1 Mark Correct solution
Q13	$D = \frac{k}{R}$ $123 = \frac{k}{7}$ $k = 861$ $D = \frac{861}{R}$	2 Marks Correct solution 1 Mark Obtains $123 = \frac{k}{7}$
Q14a	$f\big(g(x)\big) = \frac{1}{2(3x + 2) - 1}$ $f\big(g(x)\big) = \frac{1}{6x + 4 - 1}$ $f\big(g(x)\big) = \frac{1}{6x + 3}$	2 Marks Correct solution 1 Mark Shows $\frac{1}{2(3x + 2) - 1}$
Q14b	$6x + 3 \neq 0$ $x \neq -\frac{1}{2}$ $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$	1 Mark Correct solution
Q14c	$y = f\big(g(x)\big) + 1 = \frac{1}{6x + 3} + 1$ x-intercept, $y = 0$ $\frac{1}{6x + 3} + 1 = 0$ $\frac{1}{6x + 3} = -1$ $6x + 3 = -1$ $6x = -4$ $x = -\frac{2}{3}$ y-intercept, $x = 0$ $y = \frac{1}{6 \times 0 + 3} + 1$ $y = \frac{4}{3}$	3 Marks Correct solution 2 Marks Correct graph with most key features shown 1 Mark Finds the x and y intercepts

	Vertical asymptote $x = -\frac{1}{2}$ Horizontal asymptote $y = 1$  	
Q15a	Prob of both pass = Prob of Rayaan pass × Prob of Saron pass $0.533 = 0.65 \times$ Prob of Saron pass Prob of Saron pass $= 0.533 \div 0.65 = 0.82$	1 Mark Correct solution
Q15b	Prob of both fail = Prob of Saron fail × Prob of Rayaan fail Prob of both fail $= (1 - 0.82) \times (1 - 0.65)$ Prob of both fail $= 0.18 \times 0.35 = 0.063$	2 Marks Correct solution 1 Mark Finds the probability of one person failing
Q15c	Prob of at least one fail = Complement of prob of both passes $1 - 0.533 = 0.467$	1 Mark Correct solution
Q16	$y = \frac{(4 - x^2)}{e^{3x}}$ $\frac{dy}{dx} = \frac{e^{3x} \times (-2x) - (4 - x^2) \times 3e^{3x}}{(e^{3x})^2}$ $\frac{dy}{dx} = \frac{e^{3x}(-2x - 12 + 3x^2)}{e^{6x}}$ $\frac{dy}{dx} = \frac{-2x - 12 + 3x^2}{e^{3x}}$ At $(0, 4), x = 0$ $m_T = \frac{-12}{e^0}$ $m_T = -12$ $m_N = \frac{1}{12}$	3 Marks Correct solution 2 Marks Correct gradient of tangent 1 Mark Correct differentiation

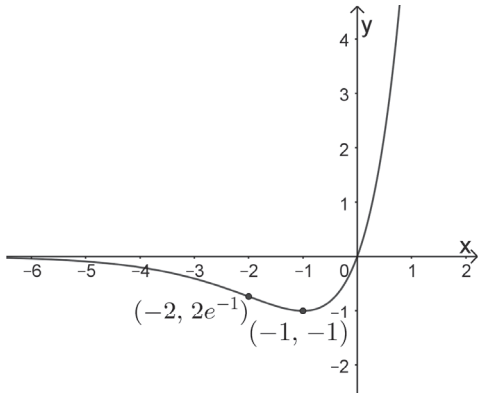
Q17a	$\int \left(6x^2 - \sin\frac{x}{3}\right) dx$ $= \frac{6x^3}{3} + 3\cos\frac{x}{3} + C$ $= 2x^3 + 3\cos\frac{x}{3} + C$	2 Marks Correct solution 1 Mark Some correct integration
Q17b	$\int_0^1 \frac{1}{(3 - 2x)^5} dx$ $= \int_0^1 (3 - 2x)^{-5} dx$ $= \left[\frac{(3 - 2x)^{-4}}{-4 \times -2}\right]_0^1$ $= \left[\frac{1}{8(3 - 2x)^4}\right]_0^1$ $= \left[\frac{1}{8(3 - 2 \times 1)^4} - \frac{1}{8(3 - 0)^4}\right]$ $= \frac{1}{8} - \frac{1}{648}$ $= \frac{10}{81}$	2 Marks Correct solution 1 Mark Correct primitive function
Q18	$T_3 = ar^2 = 21.6 \ldots\ldots (1)$ $T_5 = ar^4 = 31.104 \ldots\ldots (2)$ $(2) \div (1)$ $r^2 = 1.44$ $r = \pm 1.2$ $ar^2 = 21.6$ $a \times 1.44 = 21.6$ $a = 15$	3 Marks Correct solution 2 Marks Obtains one of the ratio and the first term 1 Mark Establishes T_3 and T_5
Q19a	$z = \frac{x - \mu}{\sigma}$ $z = \frac{28 - 34}{5}$ $z = -\frac{6}{5} = -1.2$	1 Mark Correct solution
Q19b	$1.8 = \frac{x - 34}{5}$ $x = 43$kg $43 - 34 = 9$ It needs to lose at least 9kg.	1 Mark Correct solution
Q19c	79% have a mass < 38kg $79\% - 50\% = 29\%$ ∴ 29% have a mass between 34kg and 38kg. ∴ 29% have a mass between 30kg and 34kg. 95% have a mass between 24kg and 44kg. ∴ 47.5% have a mass between 34kg and 44kg. ∴ % of mass between 30kg and 44kg is $29\% + 47.5\% = 76.5\%$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Recognises 95% is between 24kg and 44kg or finds 29% is between 34kg and 38kg

Q20a	$y = \dfrac{10}{2 \times 3 + 5\sqrt{3}}$ $y = 0.682116 \ldots$ $y \approx 0.6821$	1 Mark Correct solution
Q20b	$Area = \dfrac{1}{2}[1.4286 + 2(0.9033 + 0.6821) + 0.5556]$ $Area = 2.5775$	2 Marks Correct solution 1 Mark Correct substitution into the trapezoidal rule
Q20c	The approximation is more because the sides of the trapezia lie above the concave up curve. 	1 Mark Correct solution
Q21a	RTP: $\dfrac{1}{\sin x + 1} - \dfrac{1}{\sin x - 1} = 2\sec^2 x$ $LHS = \dfrac{1}{\sin x + 1} - \dfrac{1}{\sin x - 1}$ $LHS = \dfrac{(\sin x - 1) - (\sin x + 1)}{(\sin x - 1)(\sin x + 1)}$ $LHS = \dfrac{-2}{\sin^2 x - 1}$ $LHS = \dfrac{-2}{1 - \cos^2 x - 1}$ $LHS = \dfrac{-2}{-\cos^2 x}$ $LHS = 2\sec^2 x$ $LHS = RHS$	2 Marks Correct solution 1 Mark Shows $\dfrac{-2}{\sin^2 x - 1}$
Q21b	$\int_0^{\frac{\pi}{4}} \left(\dfrac{1}{3\sin x + 3} - \dfrac{1}{3\sin x - 3}\right) dx$ $= \dfrac{1}{3}\int_0^{\frac{\pi}{4}} \left(\dfrac{1}{\sin x + 1} - \dfrac{1}{\sin x - 1}\right) dx$ $= \dfrac{1}{3}\int_0^{\frac{\pi}{4}} 2\sec^2 x \, dx$ $= \dfrac{2}{3}\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ $= \dfrac{2}{3}[\tan x]_0^{\frac{\pi}{4}}$ $= \dfrac{2}{3}\left[\tan\dfrac{\pi}{4} - \tan 0\right]$ $= \dfrac{2}{3}$	2 Marks Correct solution 1 Mark Recognises $\dfrac{1}{3}\int_0^{\frac{\pi}{4}} 2\sec^2 x \, dx$
Q22a	$V = 375e^{0.05t}$ $V = 375e^{0.05\times 7}$ $V = 532.15 \ldots \approx \$532$ (nearest dollar)	1 Mark Correct solution

Q22b	$1200 = 375e^{0.05t}$ $\dfrac{1200}{375} = e^{0.05t}$ $\ln 3.2 = 0.05t$ $t = \dfrac{\ln 3.2}{0.05}$ $t = 23.263 \ldots \approx 23.3$ years		2 Marks Correct solution 1 Mark Shows $\ln 3.2 = 0.05t$
Q22c	$\dfrac{dV}{dt} = 375 \times 0.05e^{0.05t}$ $\dfrac{dV}{dt} = 18.75e^{0.05t}$ At $t = 7$ $\dfrac{dV}{dt} = 18.75e^{0.05\times 7}$ $\dfrac{dV}{dt} = 26.60 \ldots \approx 26.6$ The rate is \$26.6 per year	OR $\dfrac{dV}{dt} = 0.05 \times 375e^{0.05t}$ $\dfrac{dV}{dt} = 0.05 \times V$ From part (a) $\dfrac{dV}{dt} = 0.05 \times 532$ $\dfrac{dV}{dt} = 26.6$	2 Marks Correct solution 1 Mark Correct $\dfrac{dV}{dt}$
Q23a	$r = 0.99075 \ldots$ $r \approx 0.99$ The association between the mass and the length is strong and positive.		2 Marks Correct solution 1 Mark Correct r value
Q23b	$l = A + Bm$ $l = 210.6 + 0.1525 \ldots m$ $l = 210 + 0.15m$		2 Marks Correct solution 1 Mark Correct A or B value
Q23c	$275 = 210 + 0.15m$ $65 = 0.15m$ $m = 433.33 \ldots$ $m \approx 433$ grams		1 Mark Correct solution
Q24a	$r = \dfrac{0.078}{12} = 0.0065, \quad n = 4 \times 12 = 48$ From table: 41.11986 Let the monthly repayment be x. $PV = 41.11986x$ $22000 = 41.11986x$ $x = 535.021 \ldots \approx \$535.02$		2 Marks Correct solution 1 Mark Identifies 41.11986
Q24b	Total repaid $= \$535.02 \times 48$ Total repaid $= \$25680.96$ Interest $= \$25680.96 - \$22000$ Interest $= \$3680.96$		2 Marks Correct solution 1 Mark Finds the total amount to be repaid
Q25a	Let T_{n+1} be the power available after the n^{th} charge. Third charge is T_4 $T_4 = 32 \times 0.95^3$ $T_4 = 27.436 \approx 27.44$ hours		1 Mark Correct solution
Q25b	$S_\infty = \dfrac{32}{1 - 0.95}$ $S_\infty = 640$ hours		1 Mark Correct solution

Q25c	$T_{n+1} < 2$ $32 \times 0.95^n < 2$ $0.95^n < \frac{1}{16}$ $\ln 0.95^n < \ln \frac{1}{16}$ $n \ln 0.95 < \ln \frac{1}{16}$ $n > \frac{\ln \frac{1}{16}}{\ln 0.95}$ $n > 54.05 \ldots$ $\therefore n = 55$ Power is less than 2 hours after the 55th recharge.	2 Marks Correct solution 1 Mark Finds $n \ln 0.95 < \ln \frac{1}{16}$
Q26a	$5.1 + 3.6 = 8.7$ Maximum number is 870 $5.1 - 3.6 = 1.5$ Minimum number is 150	2 Marks Correct solution 1 Mark Finds max or min, or both 8.7 and 1.5
Q26b	$\frac{2\pi}{\frac{5\pi}{28}} = \frac{56}{5} = 11.2$ Period is 11.2 months	1 Mark Correct solution
Q26c	$3.6 \sin\left(\frac{5\pi}{28}t\right) + 5.1 = 3.3$ $3.6 \sin\left(\frac{5\pi}{28}t\right) = -1.8$ $\sin\left(\frac{5\pi}{28}t\right) = -\frac{1}{2}$ $\frac{5\pi}{28}t = \frac{7\pi}{6}, \frac{11\pi}{6}$ $t = \frac{98}{15}, \frac{154}{15}$  Time interval in months below 330 is $\frac{154}{15} - \frac{98}{15} = \frac{56}{15} = 3\frac{11}{15}$ Time interval in months above 330 is $11.2 - 3\frac{11}{15} = 7\frac{7}{15}$	3 Marks Correct solution 2 Marks Finds both values of t 1 Mark Finds $\sin\left(\frac{5\pi}{28}t\right) = -\frac{1}{2}$

Q27	$y = 3x^2 + 24x + 33$ $y = 3(x^2 + 8x) + 33$ $y = 3(x^2 + 8x + 16) + 33 - 3 \times 16$ $y = 3(x+4)^2 - 15$ $\therefore k = 3, p = -4, q = -15$	2 Marks Correct solution 1 Mark Finds $y = 3(x+4)^2 - 15$
Q28	$A = \int_{-1}^{\frac{3}{2}} \left(2 - x - \frac{3}{2x+3}\right) dx$ $A = \left[2x - \frac{x^2}{2} - \frac{3}{2}\ln\lvert 2x+3\rvert\right]_{-1}^{\frac{3}{2}}$ $A = \left[\left(2 \times \frac{3}{2} - \frac{\left(\frac{3}{2}\right)^2}{2} - \frac{3}{2}\ln\left\lvert 2 \times \frac{3}{2} + 3\right\rvert\right) - \left(2 \times (-1) - \frac{(-1)^2}{2} - \frac{3}{2}\ln\lvert 2 \times (-1) + 3\rvert\right)\right]$ $A = \left[\left(\frac{15}{8} - \frac{3}{2}\ln 6\right) - \left(-\frac{5}{2} - \frac{3}{2}\ln 1\right)\right]$ $A = \left(\frac{35}{8} - \frac{3}{2}\ln 6\right)\ units^3$	3 Marks Correct solution 2 Marks Correct primitive function 1 Mark Expresses area as $\int_{-1}^{\frac{3}{2}} \left(2 - x - \frac{3}{2x+3}\right) dx$
Q29a	$\int_0^{30} kx^3(30-x)dx = 1$ $\frac{1}{k} = \int_0^{30} (30x^3 - x^4)dx$ $\frac{1}{k} = \left[\frac{30x^4}{4} - \frac{x^5}{5}\right]_0^{30}$ $\frac{1}{k} = \left[\frac{30 \times 30^4}{4} - \frac{30^5}{5} - 0\right]$ $\frac{1}{k} = 1215000$ $k = \frac{1}{1215000}$	2 Marks Correct solution 1 Mark Finds $\frac{1}{k} = \left[\frac{30x^4}{4} - \frac{x^5}{5}\right]_0^{30}$
Q29b	To verify that the median is about 20.586, we need to check that $P(X \leq 20.586) \approx 0.5$ $\frac{1}{1215000}\int_0^{20.586} (30x^3 - x^4)dx$ $= \frac{1}{1215000} \times \left[\frac{30 \times x^4}{4} - \frac{x^5}{5}\right]_0^{20.586}$ $= \frac{1}{1215000} \times \left[\frac{30 \times 20.586^4}{4} - \frac{20.586^5}{5} - 0\right]$ $= 0.5000 \ldots$ ≈ 0.5	1 Mark Correct solution
Q29c	$P(X > 10) = 1 - P(X \leq 10)$ $P(X > 10) = 1 - \frac{1}{1215000}\int_0^{10} (30x^3 - x^4)dx$ $P(X > 10) = 1 - \frac{1}{1215000} \times \left[\frac{30 \times x^4}{4} - \frac{x^5}{5}\right]_0^{10}$	2 Marks Correct solution 1 Mark Finds $P(X \leq 10)$

	$P(X>10)=1-\frac{1}{1215000}\times\left[\frac{30\times10^4}{4}-\frac{10^5}{5}-0\right]$ $P(X>10)=1-\frac{1}{1215000}\times55000$ $P(X>10)=1-\frac{11}{243}$ $P(X>10)=\frac{232}{243}=0.9547\ldots$ $P(X>10)\approx0.95$	
Q30a	$g'(x)=(1+x)e^{x+1}$ $g''(x)=1\times e^{x+1}+(1+x)\times e^{x+1}$ $g''(x)=e^{x+1}(1+1+x)$ $g''(x)=(2+x)e^{x+1}$	1 Mark Correct solution
Q30b	Stationary points where $g'(x)=0$ $(1+x)e^{x+1}=0$ $x=-1$, since $e^{x+1}\neq0$ $g(-1)=-1\times e^{-1+1}=-1$ $g''(-1)=(2-1)e^{-1+1}=1>0$ $\therefore(-1,-1)$ is a minimum turning point.	2 Marks Correct solution 1 Mark Finds the stationary point $(-1,-1)$
Q30c	Point of inflection where $g''(x)=0$ $(2+x)e^{x+1}=0$ $x=-2$, since $e^{x+1}\neq0$ $g(-2)=-2\times e^{-2+1}=-2e^{-1}=-\frac{2}{e}$ x | -3 | -2 | -1 $g''(x)$ | $-e^{-2}$ | 0 | 1 Chang concavity at $x=-2$ $\therefore$ inflection point at $x=-2$ $\left(-2,-\frac{2}{e}\right)$	1 Mark Correct solution
Q30d	$g(x)=xe^{x+1}$  	2 Marks Correct solution 1 Mark Correct graph with some key features

Q31a	Initial velocity at $t=0$ $v=\frac{6}{\sqrt{5\times0+4}}$ $v=\frac{6}{\sqrt{4}}=3$ Initial velocity is $v=3$ m/s	1 Mark Correct solution
Q31b	$v=6(5t+4)^{-\frac{1}{2}}$ $\ddot{x}=\frac{dv}{dt}=6\times-\frac{1}{2}\times5\times(5t+4)^{-\frac{3}{2}}$ $\ddot{x}=-15(5t+4)^{-\frac{3}{2}}$ At $t=9$ $\ddot{x}=-15\times(5\times9+4)^{-\frac{3}{2}}$ $\ddot{x}=-15\times7^{-3}$ $\ddot{x}=-\frac{15}{343}$	2 Marks Correct solution 1 Mark Correct differentiation
Q31c	$x=\int v\,dt$ $x=\int6(5t+4)^{-\frac{1}{2}}dt$ $x=6\left[\frac{(5t+4)^{\frac{1}{2}}}{5\times\frac{1}{2}}\right]+C$ $x=\frac{12}{5}\sqrt{(5t+4)}+C$ $t=0,x=0$ $0=\frac{12}{5}\sqrt{(0+4)}+C$ $C=-\frac{24}{5}$ $\therefore x=\frac{12}{5}\sqrt{(5t+4)}-\frac{24}{5}$	2 Marks Correct solution 1 Mark Finds $x=\frac{12}{5}\sqrt{(5t+4)}+C$
Q32a	$P=2r+arcAB$ $P=2r+r\theta$ $P=r(2+\theta)$	1 Mark Correct solution
Q32b	$r(2+\theta)=22$ $r=\frac{22}{(2+\theta)}$ $A=\frac{1}{2}r^2\theta$ $A=\frac{1}{2}\times\left(\frac{22}{(2+\theta)}\right)^2\times\theta$ $A=\frac{1}{2}\times\frac{484}{(2+\theta)^2}\times\theta$ $A=\frac{242\theta}{(2+\theta)^2}$	1 Mark Correct solution

Q32c	$\frac{dA}{d\theta} = \frac{(2+\theta)^2 \times 242 - 242\theta \times 2(2+\theta)}{(2+\theta)^4}$ $\frac{dA}{d\theta} = \frac{242(2+\theta)(2+\theta-2\theta)}{(2+\theta)^4}$ $\frac{dA}{d\theta} = \frac{242(2-\theta)}{(2+\theta)^3}$ $\frac{242(2-\theta)}{(2+\theta)^3} = 0$ $2-\theta = 0$ $\theta = 2$ $\therefore$ maximum area occurs at $\theta = 2$ $A = \frac{242 \times 2}{(2+2)^2}$ $A = \frac{121}{4}\ unit^2$ This is the maximum area of the sector.	3 Marks Correct solution 2 Marks Obtains $\theta = 2$ and determines it produces a maximum area 1 Mark Correct differentiation

θ	1	2	3
$\frac{dA}{d\theta}$	$\frac{242}{27}$	0	$\frac{-242}{125}$
	/	---	\